Lecture 17: THC Theory

17.1 Preliminaries

In many previous lectures, we have merrily used variants of Stommel's box model of the THC, but we have persistently ducked the issue of why we could simply relate the THC strength to the meridional density difference. In other words, we have never explained what went into the definition of the "hydraulic" parameter, k. This avoidance has not been gratuitous, however. Rather, it reflects the difficulty of the problem, as was eloquently expressed by Alain Colin de Verdière (Colin de Verdière 1998) in his review of Joseph Pedlosky's book "Ocean Circulation Theory": "The thermohaline circulation problem, on the other hand, requires the parallel computation of both density and velocity fields and is only briefly touched upon [in Pedlosky's book]. Most recent advances on the latter topic motivated by the explosive interest in climate have come from numerical simulations and there are still many steps to be ascended on the stairway linking these numerical results and first principles."

One can add that thermocline theories, trying to explain the vertical structure of both density and velocity in the top kilometre of the ocean, share the "nonlinearity problem" with theories of the THC. What makes THC theory especially hard is that we are considering the superposition of two flow regimes, the poleward western boundary current and the equatorward interior gyre flow. The near-surface THC is the potentially small residual of large, compensating transports in these regimes. This lecture will give an introduction into THC theory, which is an active research area. Hence, we will only cover a few of the important aspects, concentrating on the dynamics (force balance) – in simplistic terms, on "what sets k?"

17.2 Early approaches: Scaling

Frank Bryan was the first to find multiple equilibria of the THC in a GCM and, in a separate paper (Bryan 1987), the first to confirm that the THC in a GCM was sensitive to the assumed degree of vertical mixing. He also presented a scaling argument for the THC strength, essentially applying to the meridional velocity an earlier derivation by Welander (1971) for the zonal flow. If the surface density increase from equator to pole, $\Delta\rho$, is assumed given, one obtains from geostrophy and thermal wind,

$$fu = -\frac{1}{\rho_0}\partial_y p\ , \tag{17.1}$$

$$f\partial_z u = g/\rho_0\, \partial_y \rho\ , \tag{17.2}$$

the scaling

$$f\frac{U}{D} = \frac{g}{\rho_0}\frac{\Delta\rho}{L}\ , \tag{17.3}$$

where U, D, and L are typical scales for zonal flow, thermocline depth, and meridional extent, respectively. Furthermore, we assume that vertical advective-diffusive balance determines thermocline depth (a nontrivial statement but perhaps defensible in the absence of wind forcing),

$$w\partial_z \rho = k_v \partial_{zz}\rho\ , \tag{17.4}$$

where k_v is vertical diffusivity. We obtain for a scaling

$$\frac{W\Delta\rho}{D} = \frac{k_v\Delta\rho}{D^2}\ , \tag{17.5}$$

or, for thermocline depth, D,

$$D = \frac{k_v}{W}\ . \tag{17.6}$$

The last equation to be used is mass conservation in the form

$$\partial_x u + \partial_y v + \partial_z w = 0\ , \tag{17.7}$$

which poses the greatest conceptual difficulties in scaling. We are interested in the zonally averaged flow, that is, a scaling for

$$\partial_y \overline{v} + \partial_z \overline{w} = 0\ , \tag{17.8}$$

where the overbar marks zonal average. But (17.8) does not contain the zonal flow any more, for which we have an expression, (17.3), based on thermal wind. A simple relationship is obtained only if one assumes horizontal isotropy, that is, the scales of zonal and meridional flows are the same. This, however, seems a poor assumption, a priori, given the aforementioned compensation of meridional flows a the same depth. Frank Bryan clearly acknowledged that he assumed, without justification, that the zonally averaged meridional flow scaled as the zonal flow, which allowed him to scale (17.8) as

$$\frac{U}{L} = \frac{W}{D}. \tag{17.9}$$

Equation (17.9) also assumes that zonal and meridional extents are comparable, and that D is the appropriate vertical scale for variations in flow as well as in stratification [cf., eq. (17.6)].

This procedure gives three scaling equations, (17.3), (17.6), and (17.9) for the three unknowns U, W, and D, which can be solved (first insert D from (17.6)) to give

$$W = \left(\frac{g \Delta\rho k_v^2}{f \rho_0 L^2} \right)^{1/3}, \tag{17.10}$$

$$D = \left(\frac{f \rho_0 L^2 k_v}{g \Delta\rho} \right)^{1/3}, \tag{17.11}$$

$$U = \left(\frac{g^2 \Delta\rho^2 k_v}{f^2 \rho_0^2 L} \right)^{1/3}, \tag{17.12}$$

which combined give the overturning scaling

$$\psi = UDL = WL^2 = \left(\frac{g \Delta\rho L^4 k_v^2}{f \rho_0} \right)^{1/3}. \tag{17.13}$$

In particular, the THC depends on the 2/3 power of vertical diffusivity, the 1/3 power of meridional density contrast, and the 4/3 power of linear basin size (because the area over which mixing can act increases). With sensible numbers ($\Delta\rho / \rho_0 = 4 \times 10^{-3}$ and

$k_v = 10^{-4} m^2 s^{-1}$), this gives 13 Sv for ψ – a remarkably good result, which might have been responsible for people looking no further. Later researchers displayed less candour than Frank Bryan, and used (17.3) directly for $\bar{v}$, without even mentioning that meridional flow is not in thermal wind balance with the meridional density gradient. Sometimes (perhaps when forced by a reviewer?) a remark was inserted that the scaling (17.13) is not particularly well founded, but it nevertheless enjoys widespread popularity.

17.3 Two-dimensional Models

For a long time, the only community that grappled – albeit indirectly – with the theoretical issue of what provides the force balance of the THC was a fairly specialist group of modellers who wanted to construct two-dimensional (latitude-depth) models of the THC, mostly for computational efficiency. The first (and simplest) of these was the one by Marotzke et al. (1988) who argued as follows. If geostrophy plus some vertical friction is assumed, the zonally averaged momentum equations are

$$-f\bar{v} = -\frac{p_E - p_W}{\rho_0 L} + A\partial_{zz}\bar{u}\ , \qquad (17.14)$$

$$f\bar{u} = -\frac{\partial_y \bar{p}}{\rho_0} + A\partial_{zz}\bar{v}\ , \qquad (17.15)$$

where A is a vertical viscosity and p_E and p_W are pressure at the eastern and western boundaries, respectively. The appearance of the boundary pressure terms presents a fundamental problem since the goal is to express everything in zonally averaged form.

The large-scale flow is nearly geostrophic, so the first two terms in each equation dominate. However, Marotzke et al. (1988) blatantly asserted that neglecting the Coriolis terms (formally assuming a nonrotating system) gave sensible results, albeit for reasons not understood. They used eq. (17.15) with f=0 to calculate the flow from the density and pressure distribution, assuming a very large A to give reasonable values. This procedure implies a simple, local and linear relationship between zonal and meridional pressure gradients, which is possible but by no means guaranteed.

Wright and Stocker (1991) constructed what is probably the best known, and arguably the most sophisticated, of all two-dimensional models. Subsequently, they devised elaborate procedures for the zonal closure (Wright et al. 1995; Wright et al. 1998). They used the results from 3-dimensional models to relate, with quite some success, zonal and meridional density differences (Fig. 17.1), and to determine the necessary coefficients. However, all attempts to justify, theoretically, the closures suffered from the difficulty that, just as Marotzke et al. (1988) had done, they never used the equation that actually determines $\bar{v}$ under three-dimensional, rotating dynamics, namely (17.14). Instead, they relied on ever more complicated versions of (17.15). The same applies to the approach of Sakai and Peltier (1995); nevertheless, a somewhat acrimonious debate ensued (Wright et al. 1998). Warren (1994) presented a

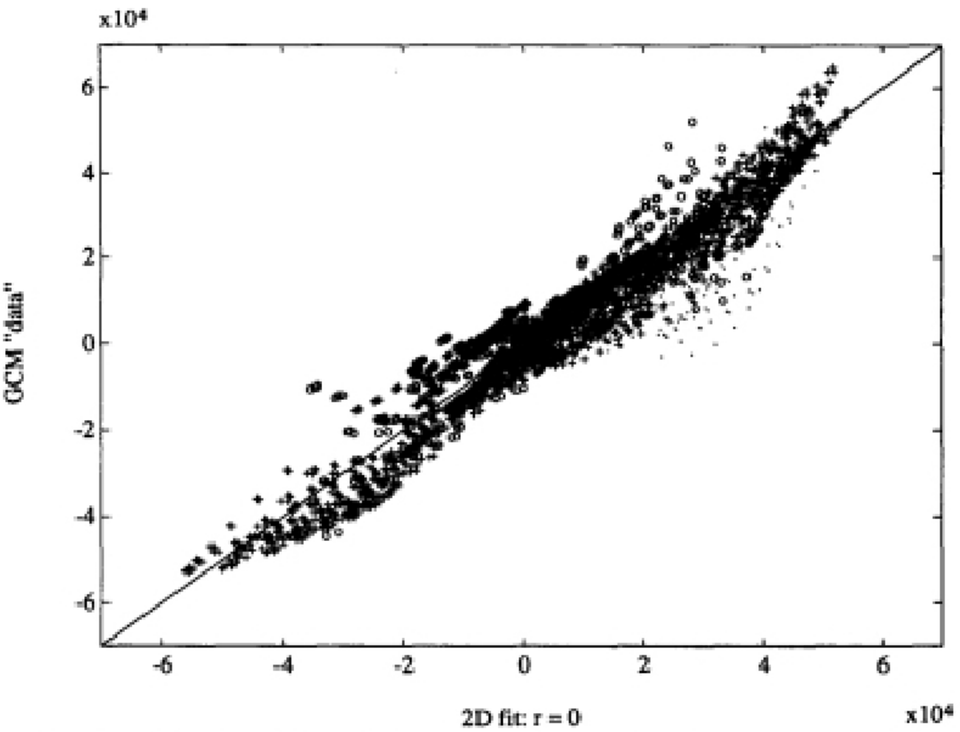

Fig. 17.1: Zonally integrated flow, in m^2s^{-1}, as implied by the 2-D closure (abscissa), against the flow in a GCM (ordinate). From Wright et al. (1995).

closure that started from Stommel-Arons theory, but he, too, asserted that the THC had to be associated with non-geostrophic (frictional) effects.

17.4 Boundary-layer Approaches

While it is not generally accepted in the community, I am convinced that the traditional two-dimensional closures represent a conceptual dead end, because they do not deal with the thermal-wind balance of the meridional flow. (To what extent they reproduce the parameter sensitivity of 3-D models still remains to be seen, on a number of important points). Instead, one should *explicitly* investigate the pressure and density distributions along the eastern and western boundaries, and calculate $\overline{v}$ from those.

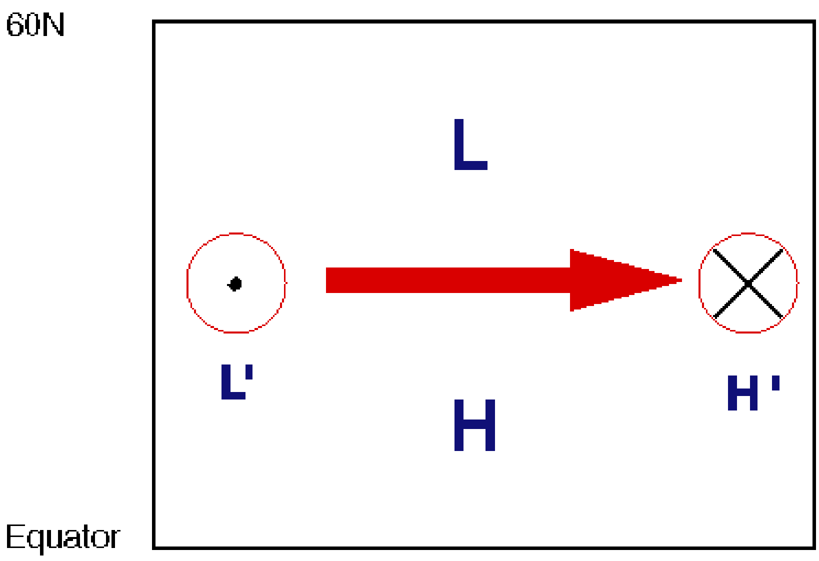

Fig. 17.2: Conceptual picture of the set-up of zonally integrated, buoyancy-driven flow.

Qualitatively, the argument goes as follows (Zhang et al. 1992; Colin de Verdière 1993). Surface waters have high density at high latitudes and low density at low latitudes. Consequently, sea level is low at high latitudes and high at low latitudes (Fig. 17.2). The resulting surface circulation is eastward. This causes a pile-up of water at the eastern boundary (a secondary high, marked H’ in Fig. 17.2), and moreover downwelling. At the western boundary, sea level would be low (marked L’), and upwelling prevails. Between H’ and L’, northward geostrophic flow ensues. A zonal section of the conceptual set-up is shown in Fig. 17.3, taken from Colin de Verdière (1993). Superimposed on the zonal overturning implied by Fig. 17.2 are the circulation and stratification of a typical subtropical gyre. Notice that neither Fig. 17.2 nor Fig. 17.3 illustrate the result of any quantitative analysis. Indeed, any attempt I undertook to solve, numerically, the equations that describe the conceptual picture of Fig. 17.3, together with advective-diffusive balance for density, have met with complete disaster. I therefore decided that a new approach was needed (Marotzke 1997).

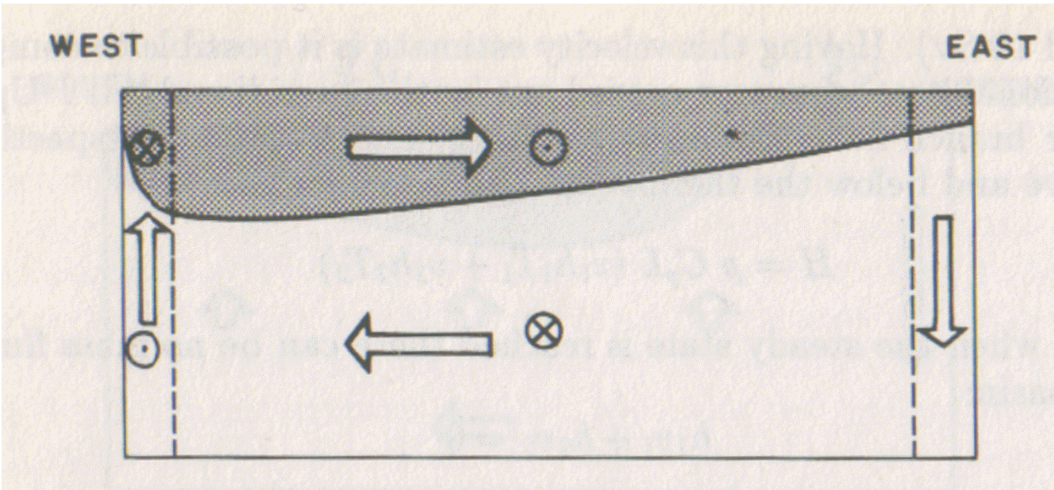

Fig. 17.3: Conceptual circulation pattern in the longitude-depth plane, corresponding to Fig. 17.2. From Colin de Verdière (1993).

Five fundamental assumptions are made, in addition to the standard approximations (hydrostatic and geostrophic balance):

i) Surface density is given and is a function of latitude only; the abyss uniformly has the properties of the densest surface water.

ii) The western boundary water is assumed to be stably stratified, following an exponential with scale height D (to be determined as part of the solution).

iii) Density in the grid cells at the lateral boundary is governed by vertical advective-diffusive balance, (17.4), except where convection is present, which then also enters the balance.

iv) Since there is no wind stress in this model, no zonal pressure gradient can be supported at the equator; in other words, isopycnals are level along the equator.

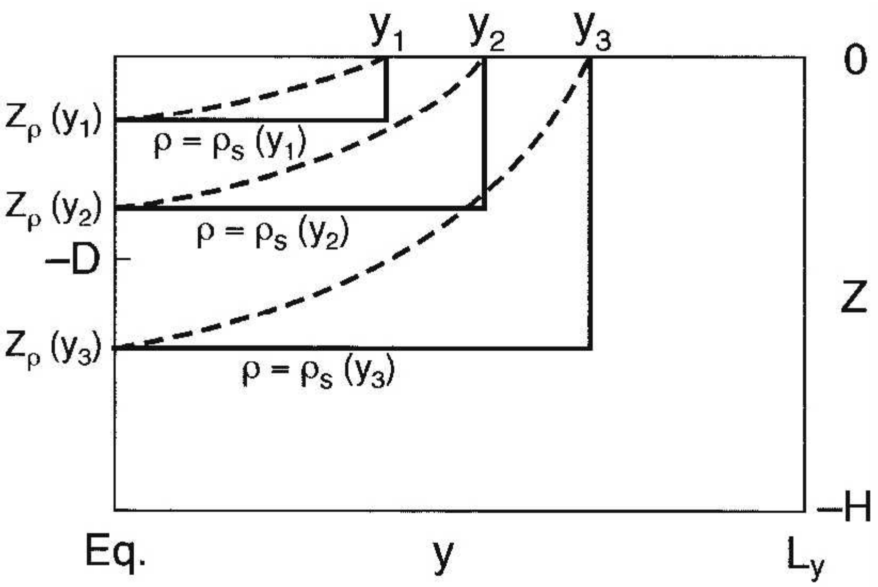

FIG. 10. Sketch of the analytical model (latitude–depth section). Shown are sample isopycnals at the eastern (solid) and western walls (dashed); notice that the model is continuous. Between east and west, there is no difference in outcrop latitudes or height (z_r) at the equator. The depth D is the exponential scale height in the western boundary.

Fig. 17.4: “Stacked boxes” underlying the THC theory. From Marotzke (1997).

v) Along the eastern boundary, convection occurs down to a depth z_ρ (to be determined as part of the solution), which is a function of latitude. In other words, the isopycnal ρ = const. is vertical at its outcrop latitude. Equatorward, it is assumed level; likewise, it is assumed that Rossby wave activity has eliminated all zonal isopycnal slopes except in the western boundary current.

Assumption *(i)* was used before by Welander (1971) and Bryan (1987); *(ii)* and *(iii)* are standard assumptions, which underlie the Bryan (1987) scaling and indirectly the Stommel-Arons picture. Assumption *(iv)* is a corollary of the force balance between wind stress and thermocline slope, traditionally assumed in equatorial oceanography. Assumption *(v)* is probably the most unorthodox; it is based on the physical picture that warm water generally moves to the northeast; subsurface advection of a certain density can only occur until the outcrop latitude is reached. That the isopycnals should be level equatorward of the outcrop latitude could be caused by Kelvin waves, but this is neither strictly required nor indeed fully confirmed by numerical experiments. Nevertheless, this "stacked boxes" concept (Jeff Scott; see Fig. 17.4) allows one to calculate the meridional overturning circulation, once the basic stratification parameter – thermocline depth, D – is known.

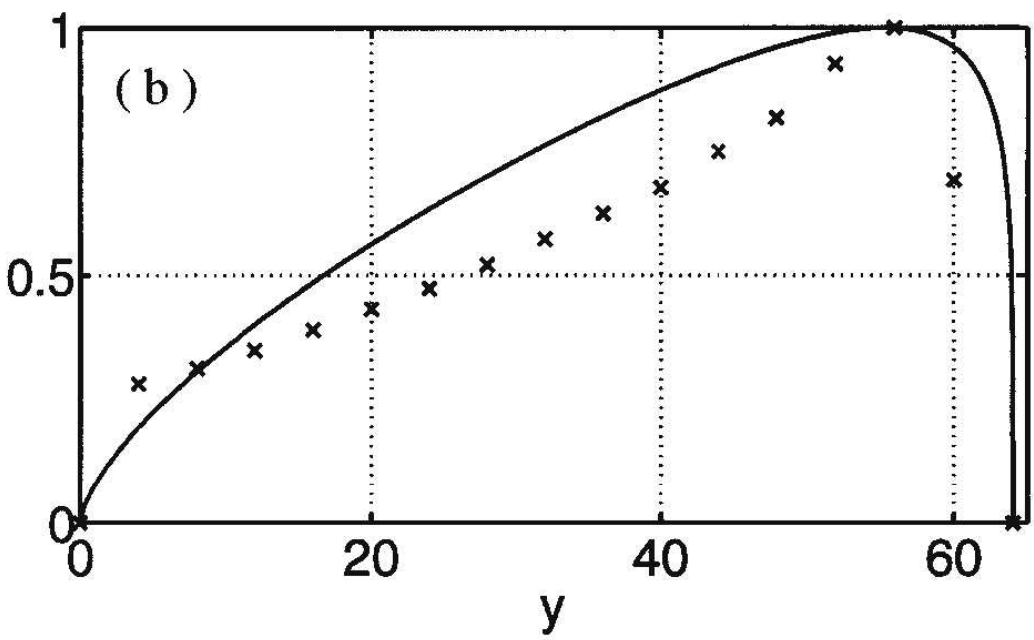

FIG. 11. (a) (Negative) depth of convection along the eastern boundary z_r divided by "thermocline depth" D, according to scaling [Eq. (6)]. (b) Latitudinal dependence of overturning according to scaling [Eq. (13), solid] and from the numerical experiment (maximum strength at every latitudinal grid point, see Fig. 1a; boundary mixing case, k_y 5 5 3 $10^{2\ 4}$ m^2 $s^{2\ 1}$; crosses).

Fig. 17.5: Theory vs. numerical experiment. From Marotzke (1997)

The logic is as follows. Assume that D is known. This implies that the density is known all along the western boundary. In particular, the depth of any isopycnal at the equator is known. As there is zero isopycnal slope along the equator, this depth, z_ρ, is known for all longitudes. It is also known along the eastern boundary, all the way northward to the outcrop latitude. Hence, density along the eastern wall is known, and one can calculate the east-west density difference. This allows us to calculate the overturning, assuming a sensible reference level (not a trivial assumption). Overall, this logical sequence allows us to calculate the flow, including w, given D. But there is a second relationship between w and D, based on the advective-diffusive balance, and both can be determined. It turns out that in this way, we can determine not only scales for w and D, but the complete dependence of overturning strength on latitude (Fig. 17.5). Indeed, the theory matches the numerical solution reasonably well.

17.5 References

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