# Lecture 9: Surface Boundary Conditions in Ocean Models

## *9.1 Surface fluxes of energy, water, and momentum*

Exchanges between ocean and atmosphere occur through two boundary layers, the marine boundary layer (MBL) in the atmosphere and the ocean's surface mixed layer (ML). To understand their interactions in detail is a research area all by itself; here, we will only scratch the surface. The mixed layer's turbulence is produced by a combination wind and buoyancy forcing. In a typical summer ML (30-50 m thick), wind-induced turbulence dominates; in winter, the ML can deepen to 600 m and more through buoyancy loss. Throughout this course, we will use the terms sea surface temperature and ML temperature synonymously, which however is only approximately correct.

The MBL's turbulence is shear-induced; the ocean is almost motionless, compared to typical wind speeds, so there is strong shear next to the surface. The high level of turbulence in the MBL causes exchanges of heat and water (evaporation). We will use dimensional arguments to obtain equations for the fluxes, the "bulk formulae" (following Gill, 1982, p. 26ff.).

The *sensible heat flux* is expected to depend on the air-sea temperature difference, the wind speed, u, and the heat capacity per unit volume of air. We can thus write

$$\begin{array}{ccccccc} H_T & = c_H & c_{p,A} & \rho_A & u & (T_S - T_A) \\ \left[\frac{W}{m^2}\right] & = & \left[\frac{Ws}{kgK}\right] & \left[\frac{kg}{m^3}\right] & \left[\frac{m}{s}\right] & [K] \end{array} , \qquad (9.1)$$

where all atmospheric variables are, by convention, calculated at 10 m height. Here, $c_H$ is a dimensionless coefficient, determined empirically and with an uncertain value that is not a universal constant but of the order of $10^{-3}$.

The evaporation rate, E', is assumed to depend on the specific humidity of air, $q_A$, (mass of water vapour per mass of air), the air density, and wind speed , all evaluated at 10 m height. Turbulence is assumed to effect an exchange between the

10-m level and the air at the surface, which is assumed to be saturated with "saturation specific humidity" $q_S$, which in turn is a function of sea surface temperature. We have, in analogy to (9.1),

$$\begin{array}{ccccc} E' = & c_E & \rho_A & u & (q_S - q_A) \\ \left[\frac{kg}{m^2 s}\right] = & & \left[\frac{kg}{m^3}\right] & \left[\frac{m}{s}\right] & \end{array} \quad . \tag{9.2}$$

The prime on E is used for later convenience; $c_E$ is again of order $10^{-3}$ but very uncertain. To obtain the latent heat flux from (9.2), E' must be multiplied by the latent heat of vaporisation, $L_V = 2.5 \times 10^6 Wskg^{-1}$.

As a rule of thumb, (9.1) gives 1 $Wm^{-2}$ per K temperature difference, for a wind speed of 1 $ms^{-1}$. A typical value for the coefficient connecting sensible heat flux and temperature difference is thus 10 $Wm^{-2}$ $K^{-1}$. Typically, latent heat loss is twice as strong as the sensible heat flux, giving an order of magnitude for the air-sea heat exchange coefficient of 30 $Wm^{-2}$ $K^{-1}$.

Surface water exchanges are more conveniently given as a velocity – 1 m/yr is a typical evaporation or precipitation rate. We will use

$$E \equiv E'/\rho_0 ; \quad P \equiv P'/\rho_0 ; \quad P - E \equiv (P' - E')/\rho_0 \tag{9.3}$$

which is in m/s, where P is precipitation. The difference between precipitation and evaporation, P-E, is a vertical velocity with which the sea surface moves up. At typically 1 m/yr or less, it is small compared to the typical "Ekman pumping" velocity of 30 m/yr, which is the driving force of horizontal gyre motions (Ekman pumping arises from the convergence of the horizontal Ekman drift, which itself results from the balance between wind stress and the Coriolis force). Because P-E is relatively small, it has negligible direct influence on mass transports ($0.5 \times 10^6 m^3 s^{-1}$, integrated over an ocean basin, compared to $30 - 100 \times 10^6 m^3 s^{-1}$ in the horizontal gyres).

In contrast, P-E has a crucial influence on the salinity budget. There is no net salt flux across the sea surface, on time-average. The change in salinity caused by P-E can be calculated in the following way (see Fig. 9.1).

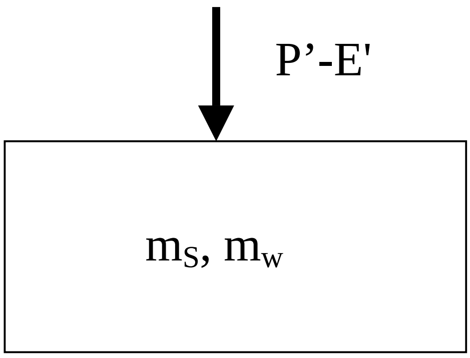


Fig. 9.1 Sketch of "bucket model"

All quantities in the sketch are given per unit area; $m_S$ is the salt mass and $m_w$ the mass of pure water in the well mixed bucket of depth D; P'-E' the net freshwater gain, that is, $m_w$ is increased per second by P'-E'. The salinity in the bucket is defined as

$$s \equiv \frac{m_s}{m_w + m_s} \tag{9.4}$$

Notice that *s* is the absolute salinity, which in the ocean has values of about 0.035. The salinity is changed by P'-E' according to

$$\frac{ds}{dt} = -\frac{m_s \dfrac{dm_w}{dt}}{\left(m_w + m_s\right)^2} = -\frac{m_s}{m_w + m_s}\frac{\left(P' - E'\right)/\rho_0}{\left(m_w + m_s\right)/\rho_0} = -s\frac{P - E}{D}\,. \tag{9.5}$$

Notice that mass of seawater per unit area is mass of seawater per unit volume times depth. By going through the same calculation but allowing for a change in salt mass, we find that

$$\frac{ds}{dt} = \frac{\left(m_w + m_s\right)\dfrac{dm_s}{dt} - m_s\dfrac{dm_s}{dt}}{\left(m_w + m_s\right)^2} = \frac{m_w \dfrac{dm_s}{dt}}{\left(m_w + m_s\right)^2}\,. \tag{9.6}$$

By equating (9.5) and (9.6), we see that the changes in salt or freshwater content lead to equivalent salinity changes if

$$\frac{dm_s}{dt} = -\frac{m_s}{m_w}\frac{dm_w}{dt}\,. \tag{9.7}$$

In other words, a pure salt flux across the surface causes the same salinity change as a freshwater input P'-E' if

$$H_s \equiv \frac{dm_S}{dt} = -\frac{m_s}{m_w}\frac{dm_w}{dt} = \frac{m_s}{\left(m_s + m_w\right)\left(1 - \frac{m_s}{m_s + m_w}\right)}\left(P' - E'\right) = -\frac{s\rho_0}{1-s}\left(P - E\right). \quad (9.8)$$

Absolute salinity is much smaller than unity, the freshwater fluxes much smaller than Ekman pumping and mass fluxes, and moreover the variation in surface salinity over the open ocean much smaller than the average value. Therefore, we will often represent the surface water exchange solely through a virtual surface salinity flux,

$$H_s = -s_0 \rho_0 \left(P - E\right), \quad (9.9)$$

where $s_0$ is a representative value. This approximation is quite good except in the vicinity of river inflow or a marginal sea such as the Baltic. The approximation (9.9) has the advantage that a globally balanced P-E field converts to a globally balanced surface salinity flux (i.e., the global mean salinity in a model does not change).

The surface momentum exchange (wind stress) can be derived from dimensional arguments as

$$\tau = c_D \rho_A u^2, \quad (9.10)$$

where $c_D$ is the drag coefficient, which is poorly determined especially at very high and very low wind speeds. Notice that wind stress appears, in this formulation, as independent of the oceanic state. However, the surface roughness (sea state) influences the drag coefficient, while wind speed influences the wave field. Apart from the surface waves, however, the wind stress is a one-way coupling in that the momentum transfer is solely determined by the atmospheric state.

Lastly, there are energy exchanges due to shortwave and longwave radiation, but they have been discussed before.

## 9.2 Thermal boundary conditions

### *9.2.1 Preliminaries*

We have stated in the preceding section how to calculated fluxes across the sea surface from the oceanic state and the atmospheric state. These prescriptions can be applied in a conceptually straightforward way to a model if the model represents both the ocean and the atmosphere. If, however, one chooses to perform climate modelling studies with an ocean-only model, be it for simplicity or for economic reasons, how should the surface fluxes be specified? In other words, what boundary conditions should be applied?

One can state a general principle, the validity of which far transcends the specific ocean-atmosphere example: *Any choice of boundary conditions is equivalent to an implicit, conceptual model of the part of the coupled system that is* ***not*** *modelled explicitly, including the interactions across the boundary.* This implies that a careful choice of boundary conditions might provide a good representation of part of the interactions of the coupled system. We will later (in the lecture about coupled box models) give a concrete example of the rather abstract principle stated here. Here, we will focus on the surface boundary condition for temperature, and will ask ourselves which choice of formulation might imply what statements about the atmosphere.

### *9.2.2 Early Approaches*

The pioneers in constructing ocean general circulation models (GCMs), Kirk Bryan and Michael Cox from the Geophysical Fluid Dynamics Laboratory (GFDL) in Princeton, relied on trial and error in their choice of surface boundary conditions (K. Bryan, pers. comm., 1992; see Marotzke, 1994). They followed practical considerations and did not look for theoretical underpinnings. The most straightforward set of boundary conditions uses heat, freshwater, and momentum fluxes from climatologies. However, Bryan and Cox obtained very unrealistic thermocline structures.

Remembering that in laboratory experiments, thermodynamic systems are easier to control by specifying temperature at the boundary (copper plate) instead of specifying the heat flux (perhaps not true any more in the age of electronic controls), Bryan and Cox simply specified *surface values* in their model, rather than *surface fluxes*. If T and S were not carried separately, they specified surface density.

This choice seems very unphysical; after all, it is the surface fluxes that drive the oceanic conditions. But notice that prescribing fluxes irrespective of the oceanic state is no more physical: The bulk formulae tell us that the sensible heat flux, for example, depends on surface air temperature, sea surface temperature (SST), and surface wind speed. The thermal surface boundary conditions should take this into account.

### *9.2.3 "Haney" boundary conditions*

Haney (1971) provided the first formulation of SST boundary conditions that was physically based. He assumed that "The ocean is in contact with an atmospheric equilibrium state which is constant in time." The upward air-sea heat exchange, H, is then written as a truncated Taylor series about the state with no air-sea temperature contrast:

$$H\left(T_O\right)=H\left(T_A\right)+\frac{\partial H}{\partial T_O}\left(T_A\right)\left(T_O-T_A\right)+\ldots \qquad (9.11)$$

where $T_O$ is SST (assumed equal to mixed-layer temperature), $T_A$ surface air temperature (assumed to be an equilibrium temperature), and $H(T_A)$ is upward net longwave plus latent heat flux minus the downward shortwave flux; no sensible heat is exchanged if $T_O = T_A$. The increase in net upward flux of the sum of longwave radiation, latent heat, and sensible heat, per degree C or K excess of $T_O$ over $T_A$, is written as $\partial H/\partial T_O$, to be evaluated at $T_O = T_A$. It is always positive.

Haney calculated the zonal averages of $H(T_A)$ and $\partial H/\partial T_O$ from data. In particular, he found that $\partial H/\partial T_O \approx 35 W m^{-2} K^{-1}$, with only 20% variation with latitude. One can rewrite (9.11) as

$$H(T_O) = \lambda_* \left(T_O - T_A^*\right), \tag{9.12}$$

with

$$\lambda_* \equiv \frac{\partial H}{\partial T_O}(T_A) \tag{9.13}$$

and

$$T_A^* \equiv T_A - H(T_A)\left(\frac{\partial H}{\partial T_O}(T_A)\right)^{-1} : \begin{cases} > T_A \text{ if } H(T_A) < 0 \text{ downward} \\ < T_A \text{ if } H(T_A) > 0 \text{ upward} \end{cases}, \tag{9.14}$$

the *apparent atmospheric equilibrium temperature*, to which $T_O$ is restored on a timescale $\lambda^{-1}$, given by

$$\lambda^{-1} \equiv \frac{c_p \rho_0 D}{\lambda_*} \sim \frac{4\times10^3 Ws\, kg^{-1} K^{-1} 10^3 kg\, m^{-3} 50m}{35 Wm^{-2} K^{-1}} \sim 6\times10^6 s \sim 60\, days\,. \tag{9.15}$$

As usual, $c_p$, $\rho_0$, and D are specific heat, reference density, and depth of the mixed layer. Notice that $T_A^*$ is neither oceanic nor atmospheric temperature. This distinction often gets blurred in applications; “restoring to observed SST” is quite a standard procedure. But notice that if one achieved perfect restoring to the data, the surface heat flux would have to be identically zero! Moreover, if a seasonal cycle is included, restoring to observations implies that the lag of modelled SST behind the solar radiation, which is typically 1-2 months, is artificially increased by $\lambda^{-1}$.

### *9.2.4 Haney conditions and atmospheric response*

Empirically, all seems fine with Haney’s (1971) SST condition. However, there are a number of conceptual problems. First, the atmosphere is unlikely to remain unchanged as SST changes. Second, atmospheric models are typically run with SST given as lower boundary conditions. Surface fluxes are calculated from the bulk formulae, which, as we saw above, could be rewritten as a restoring condition. The surface heat fluxes are hence part of the solution of the atmospheric model and could be recorded when, for example, the atmospheric model has reached a statistical steady state. But this procedure would imply that the ocean model should be driven by fluxes that are independent of the ocean model solution – in contrast to what we derived in the preceding section.

Third, prescribing SST in an atmospheric model run is justified by the larger heat capacity of the ocean, which serves as a reservoir. It is tempting to conclude the reverse in the case of a Haney condition for the ocean model – that is, one might erroneously state that the use of the Haney condition implies the assumption of a very large or infinite atmospheric heat *capacity*. This would not only be a logical fallacy (large heat capacity implies exponential damping towards reservoir conditions; the reverse could be but need not be true), but would violate all we know and would indicate that the Haney condition was a poor approximation indeed.

This is not the case, however, as shown by Davis (*J. Phys. Oceanogr.*, **6,** 249-266, 1976), who investigated non-seasonal SST and SLP (sea level pressure) variability over the mid-latitude central Pacific. He showed that SST anomalies

- Correlate with SST anomalies several months into the future (indicating their lifetime and predictability)
- Correlate with simultaneous SLP anomalies and even better with prior (1 month) SLP anomalies
- Do not correlate with future SLP anomalies.

Hence, on timescales of a month to a year, *the atmosphere drives the ocean.* A Haney condition thus seems justified empirically, but we still have to work on a theoretical understanding of why it is sensible.

### *9.3 Surface salinity boundary conditions*

Sea surface salinity (SSS) is modified through surface water exchanges. Evaporation is a function of SST, surface air temperature, and surface wind, while precipitation is a very complicated function of the atmospheric state and atmospheric processes (difficult to measure and difficult to model). The only oceanic variable that appreciably enters the SSS boundary condition is SST, but not SSS itself.

In practice, a restoring boundary condition for SSS is often used:

$$H_S = \lambda_S \left( S - S_* \right). \qquad (9.16)$$

This formulation is devoid of any physical justification and is employed solely to make the resulting salinity solution look more realistic, at any cost. Instead, one should, to lowest order of accuracy but with some physical justification, prescribe E-P or the equivalent virtual salinity flux. This would imply that all anomalous evaporation precipitates locally, and is not transported away. (Notice that merely changing evaporation does nothing – the additionally evaporated water must precipitate elsewhere to effect a change in SSS).

In the limiting case of $\lambda_S \to 0$, $S_* \to \infty$, with $\lambda_S S_* = const.$, the formulation (9.16) is equivalent to a fixed-flux law. Actually employing a fixed surface salinity flux, first by F. Bryan (1986), opened up the doors for a remarkable progress in understanding the role of the ocean circulation in climate regulation. The combination of “Haney”-type SST boundary conditions and a fixed equivalent surface salinity flux has become known as “mixed (thermohaline) boundary conditions”.